The Commutative Law of Multiplication

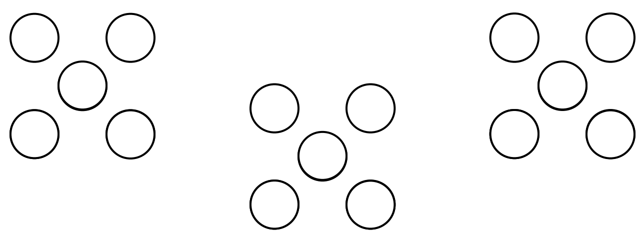
The commutative law of multiplication a simple arithmetic law that is established as part of the grade 7 curriculum. After that, we make use of it all the time without really even thinking about:

$$A \times B = B \times A$$

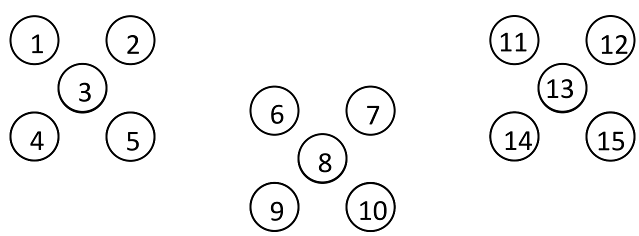
For most of us, this probably elicits a big "So What? Who Cares?"

The reason that it seems so pedantic is that most of us have actually forgotten what multiplication really is, even though we use it all the time!

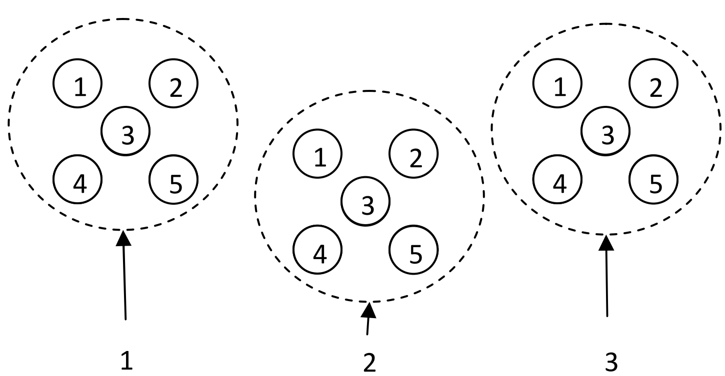
You see, fundamentally, multiplication is nothing more than a *counting* operation. It arises when we are counting multiple groups of objects; each group having the same number of individual objects. For example, consider the following diagram which represents a collection of objects:



We can count each of these objects individually as shown:



Or we can recognize that we can count a group of 5 objects three different times:

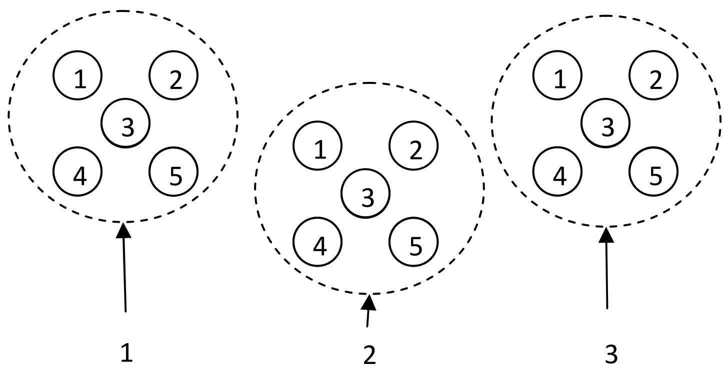


Hence we can say that the total number of objects is THREE TIMES FIVE or $5 + 5 + 5$ (or 3×5 for short)

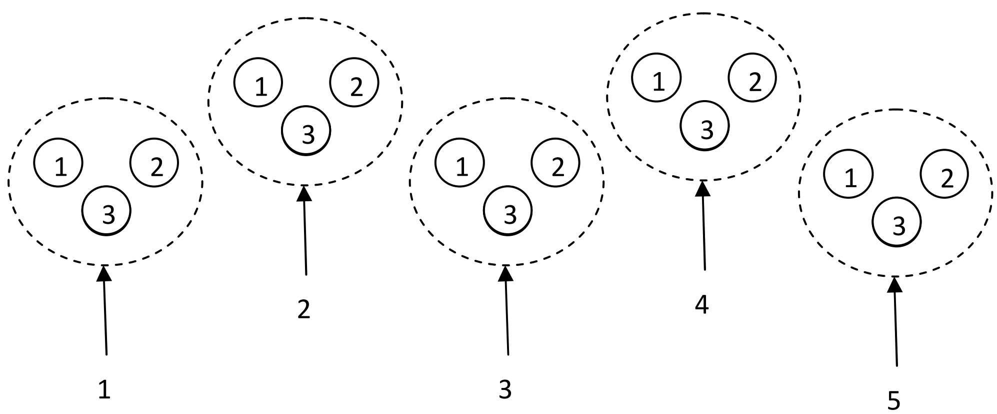
Note that this is why we use the word "TIMES" ... because we can see a group several *times*. And this is the reason why it really irks me when someone says something like "...yeah well you take the diameter and you times it by pi...". No, you don't "times it"! Anyway, back to the issue at hand:

Now that we have recalled what multiplication really means, the real surprise comes in when we recognize that THREE TIME FIVE (3×5) is *completely different* from FIVE TIMES THREE (5×3).

Behold:



3 x 5



5 x 3

So the commutative law of multiplication is actually asking us to believe that $5 + 5 + 5$ is equal to $3 + 3 + 3 + 3 + 3$. Now since these two diagrams and the corresponding expressions look completely different then I suggest to you that you should naturally be a little sceptical of the assertion that $3 \times 5 = 5 \times 3$. In this particular case we can actually do the sums to re-assure ourselves:

$$3 + 3 + 3 + 3 + 3 = 15$$

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But more than that, the commutative law says that this isn't just a fluke, but that it works for *any* two numbers. Even say, 17 and 131. Now, do you really believe that:

$$17 + 17 + 17 + \cdots + 17 \qquad \text{(one hundred and thirty-one terms)}$$

Is equal to:

$$131 + 131 + 131 + \cdots + 131 \qquad \text{(seventeen terms)}$$

???

I think that you should be pretty astonished if the sums really do add up. After all, the numbers 17 and 131 seemingly have nothing in common! Well, I used a spreadsheet to confirms that both sums equal 2227!

So the next time that you just go ahead and write $A \times B = B \times A$ perhaps you'll not take it so lightly.

The commutative law of multiplication. Can *you* prove it?

I'll present a proof in another post.